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Tropospheric Effect on the Transmissivity of the Spacecraft–Ground Tracking Station Communication Line

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Abstract—Algorithms ensuring high-speed and reliable data transmission under lognormal amplitude fluctuations (described by Fraunhofer diffraction) in the spacecraft–ground tracking station line for coherent and incoherent reception of signals have been considered. An advantage of the coherent reception of millimeter-range signals with a random error-correcting code has been indicated.

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INTRODUCTION

As shown in [1], the use of the millimeter (mm) range significantly increases (due to the increased bandwidth of the data transmission channel) the spectral efficiency and transmissivity of wireless data transmission ground–space radio interferometry (GSR) in a spacecraft–ground tracking station (SC–GTS) line up to a rate that is comparable with the rate of digital broadband data recording in the onboard memory of spacecraft, thus ensuring the continuous operation of a ground–space interferometer.

When propagating in a turbulent atmosphere, the lognormal fluctuations of amplitude $A(t)$ of millimeter (mm) and submillimeter radiowave signals can be expressed through normally distributed level χ [2] with a zero mean. Level χ corresponds to the logarithm of normalized wave amplitude: $\chi = \ln\left(A(t)/A_0\right)$. Experimental data are well consistent with the normal distribution of random variable χ in cases when the first approximation of the method of smooth perturbations is applicable [2]:

$$C_\varepsilon^2 k^{1/3} z \ll 1, \qquad (1)$$

where C_ε is the structural constant (the structural function of dielectric permeability); k is the wavenumber ($2\pi/\lambda$), where λ is the wavelength; z is the path length of an electromagnetic wave along a channel with lognormal fluctuations (along a tropospheric channel); and $A(t)$ and A_0 are the instantaneous wave amplitude and wave amplitude in an undisturbed medium, respectively. For $C_\varepsilon = 0.5 \cdot 10^{-6}$ m$^{-2/3}$, $\lambda = 0.004$ m, and $z < 400 \cdot 10^3$ m, this inequality is satisfied. Since the amplitude logarithm and level χ are distributed normally, amplitude $A(t)$ and normalized amplitude $X(t) = A(t)/A_0$ have a logarithmically normal distribution.

It should be noted that the probability density of the signal-to-noise ratio (SNR) of mm-range signals for an atmospheric channel is also described by a lognormal law [3, 4], and its variance, along with the wavelength, depends on the propagation length in the troposphere, which in turn depends on the antenna elevation angle [1, 2]. The probability density of the SNR can be expressed as [3]

$$p(\gamma) = \left(1\Big/2\gamma\sqrt{2\pi\sigma_\chi^2}\right)\exp\left[-\left(\ln\sqrt{\gamma/\gamma_0} + \sigma_\chi^2\right)^2\Big/2\sigma_\chi^2\right], \qquad (2)$$

where σ_χ^2 is the variance of the lognormal process, and γ and γ_0 are the instantaneous and mean values of the SNR at the receiver input, respectively.

VARIANCE OF LOGNORMAL FLUCTUATIONS AS A FUNCTION OF PHYSICAL PARAMETERS OF THE TROPOSPHERE

The variance in (1) is determined from the dependence of the ratio of the radius (R) of the first Fresnel zone

$$R = \sqrt{\lambda z_1}, \qquad (3)$$

to internal and external turbulence scales [2, 5]. In (3), z_1 is the total length of the electromagnetic wave propagation path from the transmitting antenna. It is known [5] that the internal scale of turbulence (l_0) depends on kinematic viscosity of air (ν) by formula

$$l_0 = \sqrt[4]{\nu^3/\varepsilon} \qquad (4)$$

and has a dimension order in the surface layer of around 1 mm. In (4), ε is the turbulence energy dissipation rate [5].

The external scale of turbulence (L_0) depends on turbulent eddies described by the Kolmogorov–Obukhov law for isotropic media and is caused by the nonuniformity of air heating. The order of magnitude of L_0 corresponds to the dynamic range (L_0/l_0) of turbulence [5] 10^3–10^4 and is approximately 10 m in the surface layer.

First, we consider the case when the radius of the first Fresnel zone is much smaller than the internal scale of turbulence ($R \ll l_0$). Since the logarithm of the amplitude and level χ are distributed normally with a zero mean, the mean square of χ is equal to its variance $\left(\overline{\chi^2} = \sigma_\chi^2\right)$. It is known from [2] that if $R \ll l_0$, we have

$$\left\langle \chi^2 \right\rangle = \sigma_\chi^2 = z^3/24 \int_0^\infty \left[\Delta_\perp^2 \psi_\varepsilon(\rho,\zeta)\right]_{\rho=0} d\zeta, \tag{5}$$

where $\psi_\varepsilon(\rho,\zeta)$ is the autocorrelation function of complex field ζ and $\Delta_\perp$ is the operator of transverse (with respect to the propagation path z) differentiation; a differential operator in the linear space of smooth functions. In this case, the variance is determined by the method of geometric optics, depending on random focusing–defocusing (lensings) of objects with a size of the order of l_0. It follows from (5) that for $R \ll l_0$ [2], the variance increases cubically depending on the distance. At a wavelength of around 4 mm, the radius of the first Fresnel zone (R) significantly exceeds the internal scale of turbulence in the propagation path of an electromagnetic wave. Here, the turbulence energy dissipation rate in the troposphere can increase a hundredfold with an increase in height [5]; however, according to (4), this can lead to an increase in l_0 only by 3.16 ($\sqrt[4]{100}$) times.

Now, we consider the situation when the radius of the first Fresnel zone considerably exceeds the internal scale of turbulence and is much smaller than the external scale: $L_0 \gg R \gg l_0$. In this case, the focusing effect by objects with a size of the order of l_0 described by the method of geometrical optics has little influence. For this case, the characteristic contribution in determining the variance is given by the Fresnel diffraction [2] or even geometric optics from objects of dimension L_0, rather than by the Fraunhofer diffraction. The variance for this case [2] is determined by the following formula:

$$\left\langle \chi^2 \right\rangle = \sigma_\chi^2 = \psi_\chi(0,z) = N C_\varepsilon^2 k^{7/6} z^{11/6}, \tag{6}$$

where N is a numeric constant [2]: $N = \pi^2 A/2 \int_0^\infty \left(1 - \sin t^2/t^2\right) t^{-8/3} dt \approx 0.077$ and A is a constant multiplier equal to 0.033. It follows from (6) that the mean square fluctuation of the amplitude (variance) at $L_0 \gg R \gg l_0$ increases with distance almost quadratically.

Finally, we consider the case when the radius of the first Fresnel zone is much larger than the external scale of turbulence: $R \gg L_0$. Here, the variance is determined as [2]

$$\left\langle \chi^2 \right\rangle = \sigma_\chi^2 = \sqrt{2\pi}/8\, \sigma_\varepsilon^2 k^2 a z \left(1 - \mathrm{arctg} D/D\right), \tag{7}$$

where σ_ε^2 is the variance of fluctuations in the dielectric permeability, a is a parameter (radiation aperture size) characterizing the inhomogeneity of the field of the external scale of turbulence (L_0), and D is wave parameter:

$$D = 2\pi\lambda z_1/l_0^2. \tag{8}$$

It is known that at $D \gg 1$, in the case of Fraunhofer diffraction, and within the first Fresnel zone with radius $\sqrt{\lambda z_1}$, there are many field inhomogeneities [2] corresponding to the correlation radius ~~L_0~~ of the fluctuation of dielectric permeability [5] for the external scale of turbulence ~~(l_ε)~~. Therefore, according to the central limit theorem of the probability theory, the distribution a approaches to the normal distribution. The normalization of these quantities is due to the "filtering" action of free space and has the same nature as the normalization of temporal signals at the output of narrow-band filters [2]. Accordingly, the correlation function of the fluctuation of dielectric permeability in the propagation medium under condition $R \gg L_0$ is described by a Gaussian curve:

$$\psi_\varepsilon(r) = \sigma_\varepsilon^2 \exp\left(-r^2/2a\right), \tag{9}$$

here, the size of inhomogeneities is characterized by the single scale a.

It follows from (8) that the wave parameter is proportional to the ratio of the squares of the radius of the first Fresnel zone and the internal scale of turbulence and increases linearly with z_1. In this case, if $R \gg L_0$, the Fresnel diffraction effect is small and Fraunhofer diffraction is prevalent. When the radius of the first Fresnel zone is comparable with the internal or external scales of turbulence, the influence of geometric optics and Fresnel diffraction or Fresnel diffraction in combination with Fraunhofer diffraction is significant.

For sufficiently large values of z ($z \gg L_0$), the structure function [2] is known to undergo saturation and be equal to doubled variance of dielectric permeability:

$$D_\varepsilon(z_1) = D_\varepsilon(\infty) = C_\varepsilon^2 L_0^{2/3} = 2\sigma_\varepsilon^2. \tag{10}$$

In view of (10) and if $D \gg 1$, one can neglect the term arctan(D/D) in (7); then, the average square of level χ will be

$$\left\langle \chi^2 \right\rangle = \sigma_\chi^2 \approx \sqrt{2\pi}/16\, C_\varepsilon^2 L_0^{2/3} k^2 a z. \tag{11}$$

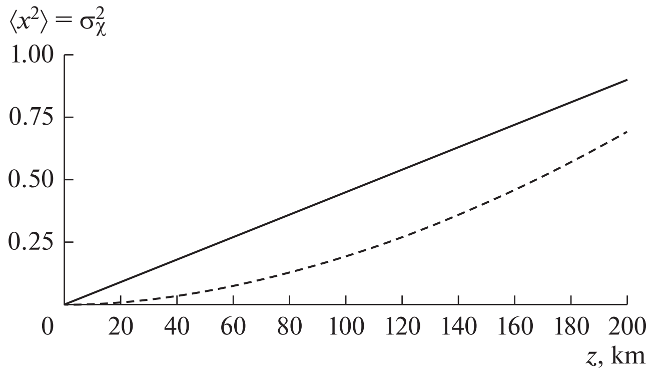

Fig. 1.

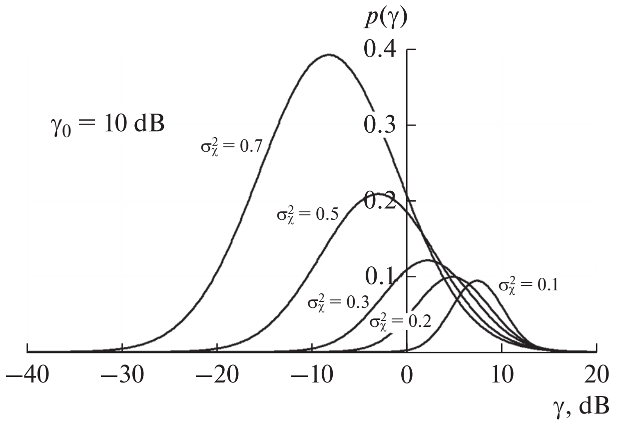

Fig. 2.

Inhomogeneity scale a is proportional to the correlation radius of the fluctuation in dielectric permeability and is always smaller than the vortices of the external scale of turbulence (L_0). We assume that $a = L_0$ for limiting from above the mean square of χ. In view of this assumption, the variance of the lognormal process is

$$\left\langle \chi^2 \right\rangle = \sigma_\chi^2 \approx \sqrt{2\pi}/16\, C_\varepsilon^2 L_0^{5/3} k^2 z. \tag{12}$$

It follows from (12) that for $R \gg 1$, the mean square of χ depends on the length of the electromagnetic wave path along the tropospheric channel (z) linearly. Figure 1 shows this mean square (variance) as a function of the length of the electromagnetic wave path along the tropospheric channel (z) at $R \gg L_0$ (solid line) and at $L_0 \gg R \gg l_0$ for $\lambda = 4$ mm.

It can be seen from the graph and Eq. (12) that the variance of χ in the initial part of the path length (z) at $R \gg L_0$ significantly exceeds the variance for the case when the radius of the first Fresnel zone differs significantly from the external and internal scales of turbulence ($L_0 \gg R \gg l_0$) because the coefficient (wavenumber) k^2 for such a small wavelength is large despite the linear dependence on z. For example, the variance at $z = 40$ km is almost 0.18.

For z_1 of almost $1.5 \cdot 10^9$ m (Lagrange L2 point) [1] and a wavelength of 4 mm, the radius of the first Fresnel zone near Earth's surface is around 2450 m, which is significantly larger than the external scale of turbulence (L_0). Therefore, in this case, Fraunhofer diffraction ensures a rapid increase in the variance of χ with z.

PROBABILITIES OF ERRONEOUS RECEPTION OF PHASE-SHIFT SIGNALS IN THE TROPOSPHERIC CHANNEL AT INCOHERENT AND COHERENT DEMODULATION

Expression (2) describes the probability density of the instantaneous value of SNR in the tropospheric channel. Figure 2 shows this density for an average SNR (γ_0) of 10 dB and different values of the variance. The analysis of the curves shows that as the variance increases, the probability of the instantaneous value of SNR shifts to a range of low values.

By averaging the error probabilities in Gaussian noise in terms of statistics of lognormal fadings in the tropospheric channel, we calculate the error probability for incoherent (13) and coherent (14) methods of signal reception (OFM-2 and FM-2/FM-4, respectively) depending on the average value of SNR (γ_0)

$$P_{nc}(\gamma_0) = \left(1\Big/4\sqrt{2\pi\sigma_\chi^2}\right) \times \int_0^\infty 1/\gamma \exp\left[-\left(\ln\sqrt{\gamma/\gamma_0} + \sigma_\chi^2\right)^2 \Big/ 2\sigma_\chi^2\right] \exp(-\alpha\gamma)\, d\gamma, \tag{13}$$

$$P_c(\gamma_0) = \left(1\Big/4\sqrt{2\pi\sigma_\chi^2}\right) \int_0^\infty 1/\gamma \times \exp\left[-\left(\ln\sqrt{\gamma/\gamma_0} + \sigma_\chi^2\right)^2 \Big/ 2\sigma_\chi^2\right] \operatorname{erfc}\sqrt{\alpha\gamma}\, d\gamma, \tag{14}$$

where $\alpha = 1$ for phase-shift signals. The dependences of the error probability for incoherent and coherent reception of signals at different variances are shown in Figs. 3 and 4, respectively.

The incoherent reception of signals differs from coherent reception by a simple demodulator with no identification of a carrier frequency by complex patterns using a narrowband bandpass filter [6, 7]. The bit/character value is determined by comparing n and $n + 1$ characters. In addition, an incoherent demodulator is less inertial, when a signal appears fairly quickly, starting from the second symbol, it is demodulated [6].

The coherent reception of signals is provided by identifying the carrier of the coherent base, with respect to which the phase of the received signal is identified [6, 7]. This demodulator is more complex and inertial because the identification mechanism of the carrier of base frequency contains a narrowband band-pass filter that filters the received signal from

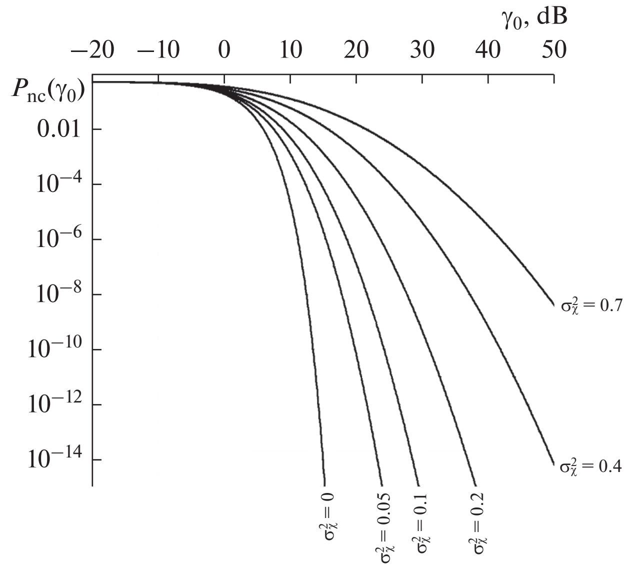

Fig. 3.

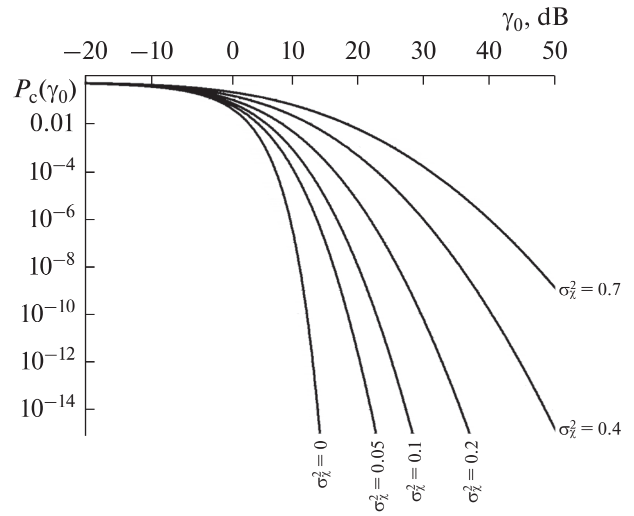

Fig. 4.

noise, which in turn ensures enhanced noise-~~resistance~~ (lower error probability at a given SNR). The narrow band of the filter governs the accuracy of the coherent base identification; here, additional requirements are imposed on the data transmission channel [6].

The correlation interval of the signal amplitude and phase fluctuations is determined by the change in dielectric permeability affecting variations in the refractive index [5] and ranges from units to tens of seconds. In view of this, the data transmission channel is relatively quiet with no significant fast fadings. In this case, it makes sense to use a coherent demodulator, which, despite its relative complexity, is more noise-~~resistant~~ than the incoherent modulator.

For example, for an SNR of 9.29 dB (at a data transmission rate in the simplex of up to 16–20 Gbit/s) obtained in [1] for the case of a transmitter located at Lagrange point L2 ($1.5 \cdot 10^9$ m), at an antenna elevation angle of 17.3° and a path length along the tropospheric channel (z) is 40 km, the variance (σ_χ^2) is around 0.2 (Fig. 2). Under these conditions, the error probability of incoherent and coherent receptions is 0.024 and $9.66 \cdot 10^{-3}$, respectively In the second case, the use of random noise-~~resistant~~ Euclidean geometry Low-Density Parity Check (LDPC) code with linear extension and iterative Sum-Product Algorithm (SPA) decoding [8] makes it possible to reduce the error probability to 10^{-5}.

When a spacecraft shifts from the vicinity of L2 to the *Radioastron* geocentric orbit, the average SNR at the receiver input increases by at least 12 dB. The radius of the first Fresnel zone still substantially exceeds the external scale of turbulence even in the perigee, ensuring that Fraunhofer diffraction conditions are met and the variance of lognormal fluctuation along the tropospheric path length increases linearly. In this case, for $\sigma_\chi^2 = 0.2$, the coherent demodulator provides an error probability no worse than $5 \cdot 10^{-6}$, even without the use of an error-resistant code.

The probability density of the lognormal process for the fluctuation in the amplitude and instantaneous value of the SNR is obtained for the case when the first approximation of the method of smooth perturbations is applicable [2]; here, the experimental data are well consistent with theoretical data up to the variances $\sigma_\chi^2 \leq 1$. This is satisfied in the examples presented in Fig. 1.

At present, along with isotropic Kolmogorov (referred to as incoherent) turbulence, coherent turbulence is actively studied [9]. The spectrum of coherent turbulence is narrower and rapidly decomposing with respect to the spectrum of the incoherent structure. Due to this, coherent turbulence is a three-dimensional topological soliton ranging from a single ordered Bénard cell to systems that are periodically distributed hydrodynamic perturbations in space (such as systems of various shafts). Here, the largest systems with a radius of up to 5000 km are Ferrel and Hadley cells [9]. They can be considered as a type of Bénard cells in a thin spherical layer (on Earth's scale). In this type of coherent turbulence, the condition for normality of distributed level χ is no longer satisfied and the parameters of a random signal in this type of turbulence should be determined experimentally using statistical methods by samples of random variables [10].

CONCLUSIONS

The use of the mm range in the GSR in the absence of tropospheric hydrometeors significantly increases the rate of data transmission in the SC–GTS line due to the increased channel frequency range. However, signal amplitude fluctuations emerging from turbulence of the troposphere reduce the noise-~~resistance~~ and data transmission rate. These factors govern the following specific features of using mm-range radiowaves in the arrangement of communication channels: the antenna elevation angle should exceed 17°; otherwise, the radio signal path in the troposphere significantly increases, which increases the variance of log-normal amplitude fluctuations on the one hand and increases the signal attenuation on the other hand. The resulting mechanism of data transmission seems to be flexible; for example, the transmission rate can be reduced from 16–20 to 8–10 Gbit/s, which still is a sufficiently high rate, but the SNR at the receiver input increases by 3 dB. Without reducing the total transmission rate, one can reduce the information rate, thus applying a modern noise-resistant code, which varies depending on the external conditions during the long-term operation of the GSR; when soliton-type coherent turbulence structures occur along the radio-signal propagation path with level χ not distributed normally, the statistical parameters of the signal should be determined experimentally.

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