



Images of black holes

Vyacheslav Dokuchaev¹ and Natalia Nazarova²

¹INR RAS, Moscow

²SISSA, Trieste, Italy

Astro Space Center, Lebedev Physical Institute — 2021, Moscow

Shapes of black hole images depend on the distribution of emitting matter around black holes

Astrophysical Case 1: radiation outside photon spheres

Luminous stationary background behind the black hole

Classical black hole shadow is viewed, which is a capture photon cross-section in the black hole gravitational field

Astrophysical Case 2: radiation inside photon spheres

Luminous accretion inflow near the black hole event horizon

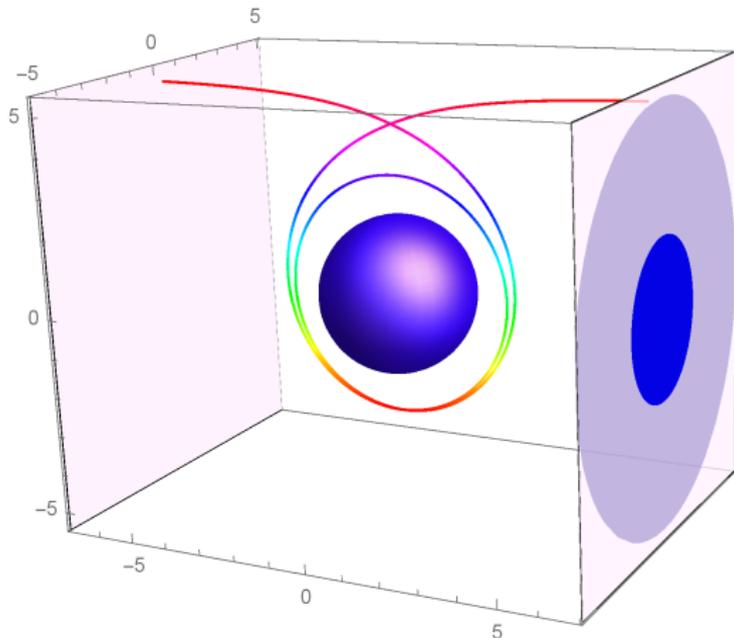
Event horizon shadow is viewed, which is a lensed image of the event horizon globe

Astrophysical Case 1: Stationary background

Euclidean image (blue disk) of the event horizon, $r_h = 2$, $M_h G/c^2 = 1$

Classical shadow (magenta region) of the Schwarzschild black hole ($a = 0$),

shadow radius $r_{sh} = 3\sqrt{3} \approx 5.2$, radius of photon circular orbit $r_{ph} = 3$

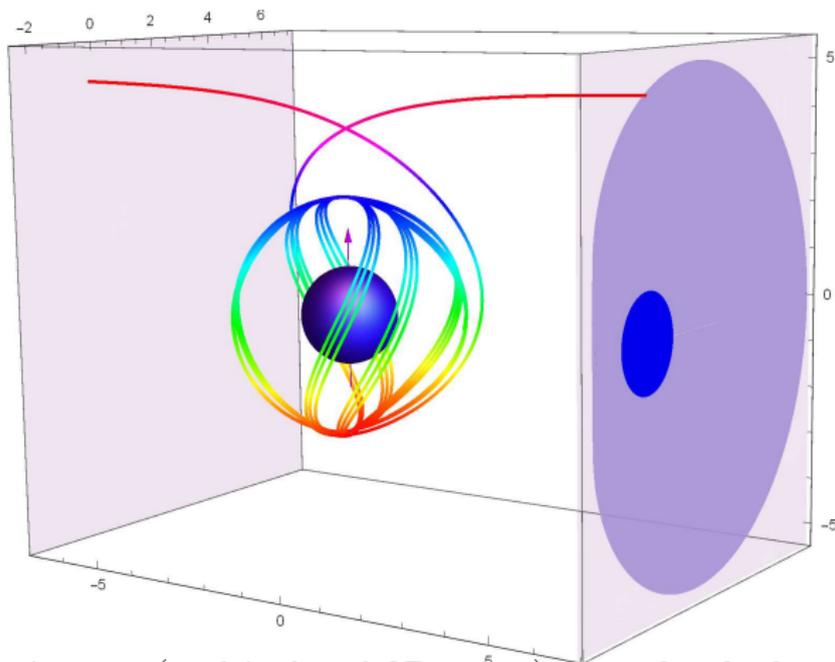


Photon trajectory (multicolored 3D-curve) with the return point

$r_{\min} = r_{ph} = 3$, impact parameters $\lambda = 0$, $q = \sqrt{Q} = r_{sh} = 3\sqrt{3} \approx 5.2$

Astrophysical Case 1: Stationary background

Classical shadow (magenta region) of the Kerr black hole: $a = 1$, $r_h = 1$
 $\theta_0 = \pi/2$ — polar angle of a distant observer



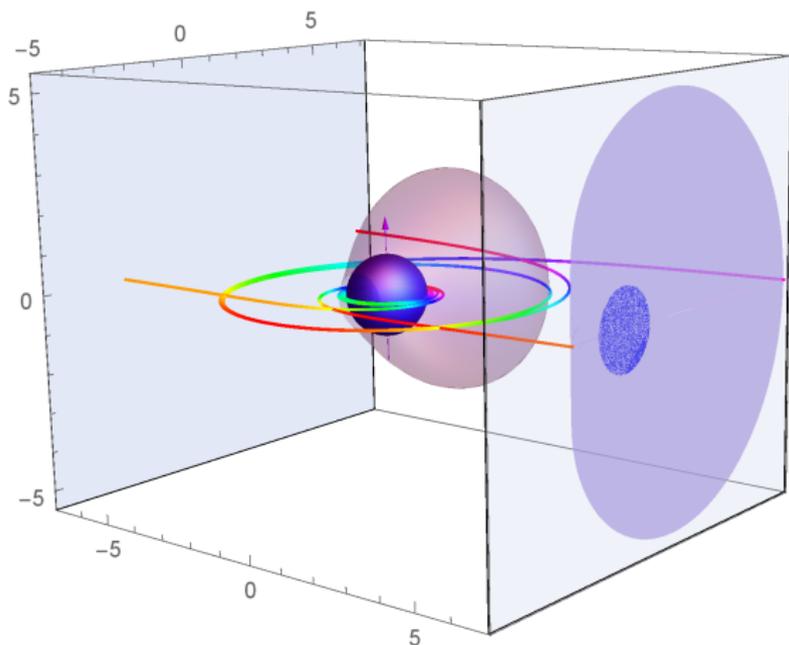
Photon trajectory (multicolored 3D-curve) near the shadow outline with the return point $r_{\min} = 1 + \sqrt{2}$ and impact parameters

$\lambda = 0$ (horizontal) and $q = \sqrt{Q} = \sqrt{11 + 8\sqrt{2}} \approx 4.72$ (vertical)

Astrophysical Case 1: Stationary distant background

(Radiation outside purple photon spheres r_{ph} at $a = 1$)

Classical black hole shadow is viewed: magenta region r_{sh}

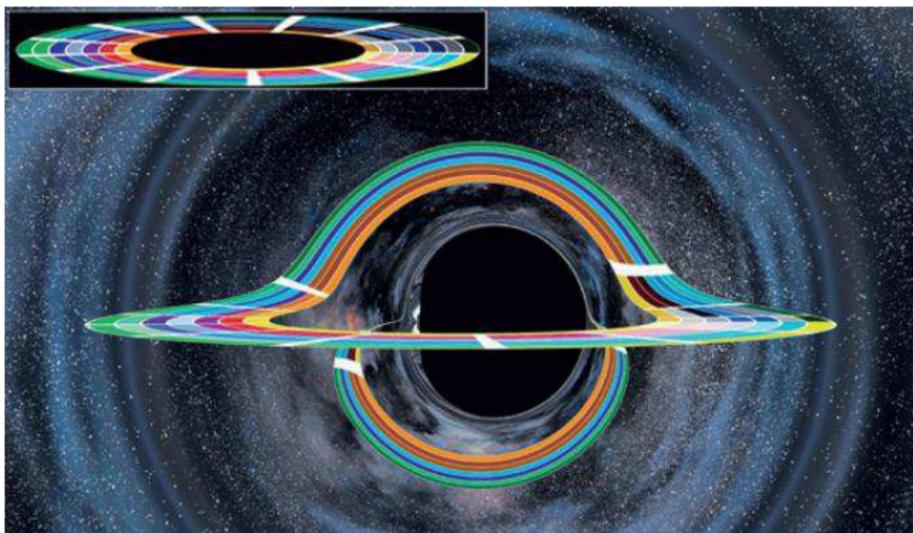


Photon trajectories (multicolored 3D-curve) near the shadow outline with the return points at $r_{min} = 1$ (co-rotating) and $r_{min} = 4$ (counter-rotating).

Closed purple region – boundary of photon spheres

Astrophysical Case 1: Stationary distant background

Modeling for the cinema “Interstellar” K.S.Thorne et al. 2015



Crucial features were neglected by demand of the cinema producer:

- (1) Doppler effect and gravitational red-shift of photons!
- (2) Emission of accretion disk near the event horizon!

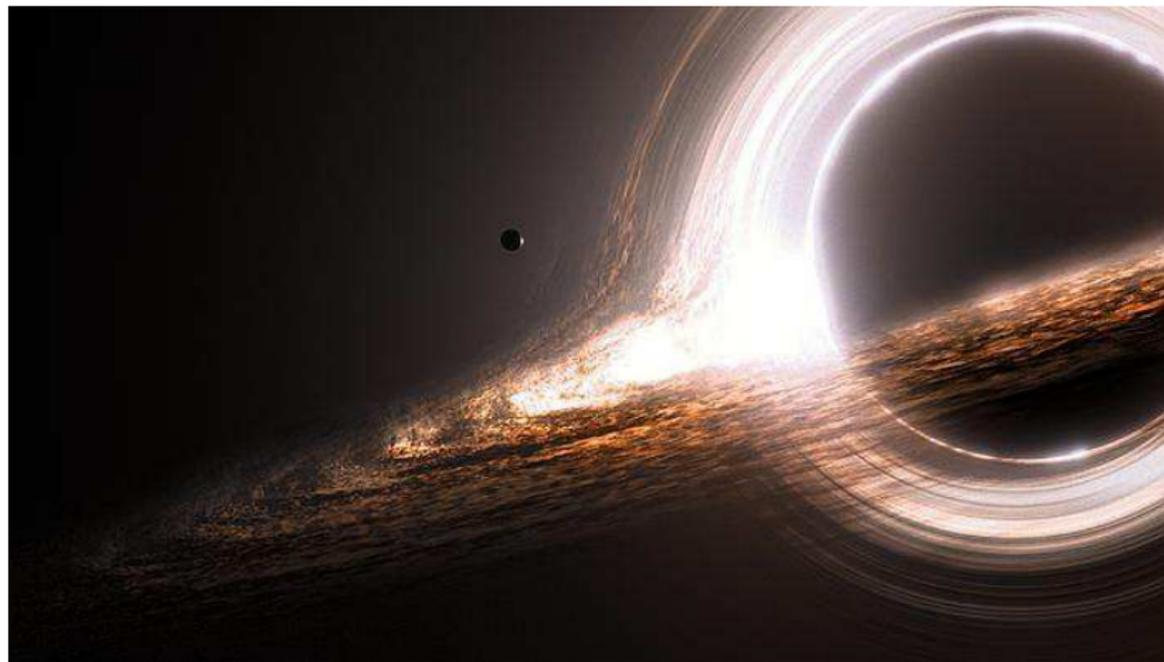
$r_{\text{int}} = 9.26$ — internal border of the disk!?

(Radiation **only outside** the photon spheres r_{ph} !)

In result, it is seen the dark classical black hole shadow

Interstellar

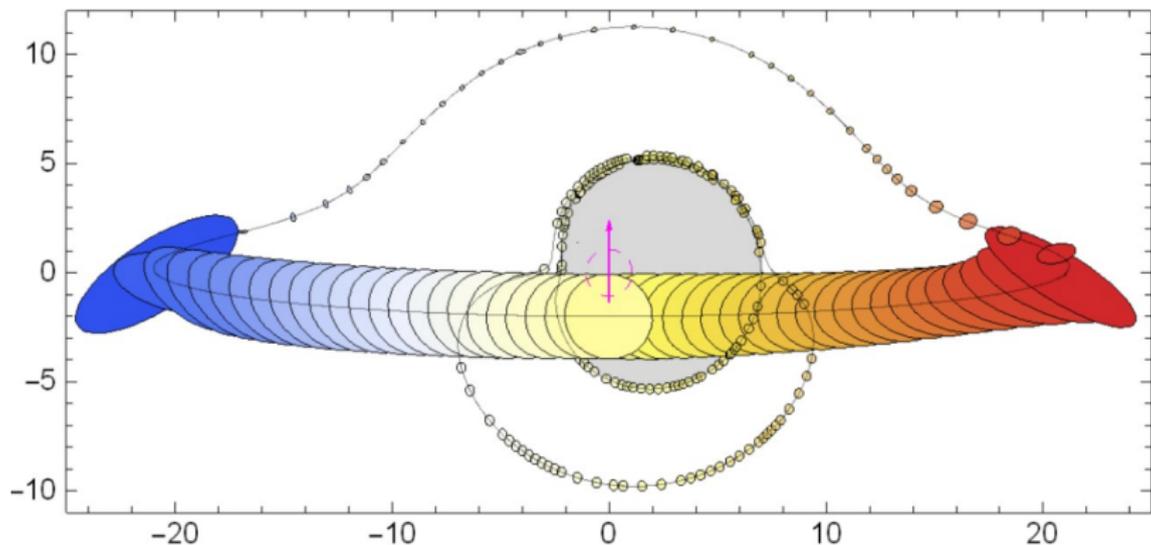
Classical black hole shadow inside accretion disk



This black hole image is wrong!

Astrophysical Case 1: Compact star on the equatorial circular orbit with radius $r_s = 20M_h G/c^2$ around SgrA*, observed by a distant telescope (Millimetron)

Radiation **outside** the photon spheres r_{ph}

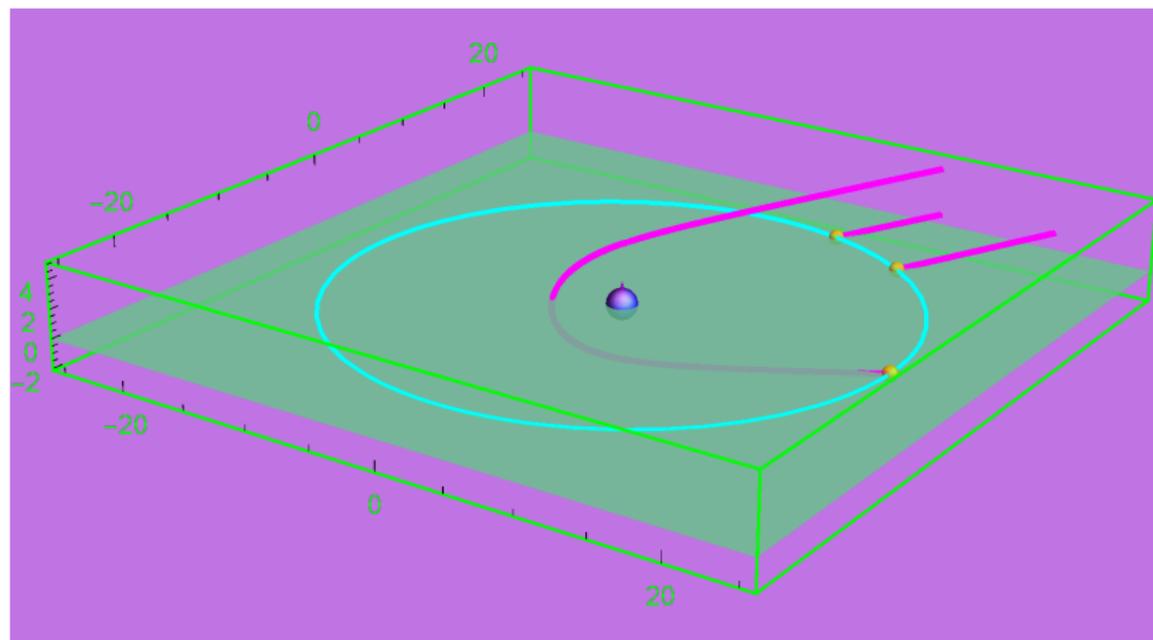


VD, Natalia Nazarova arXiv:1802.00817

3D photon trajectories

Prime image: no intersections of equatorial plane

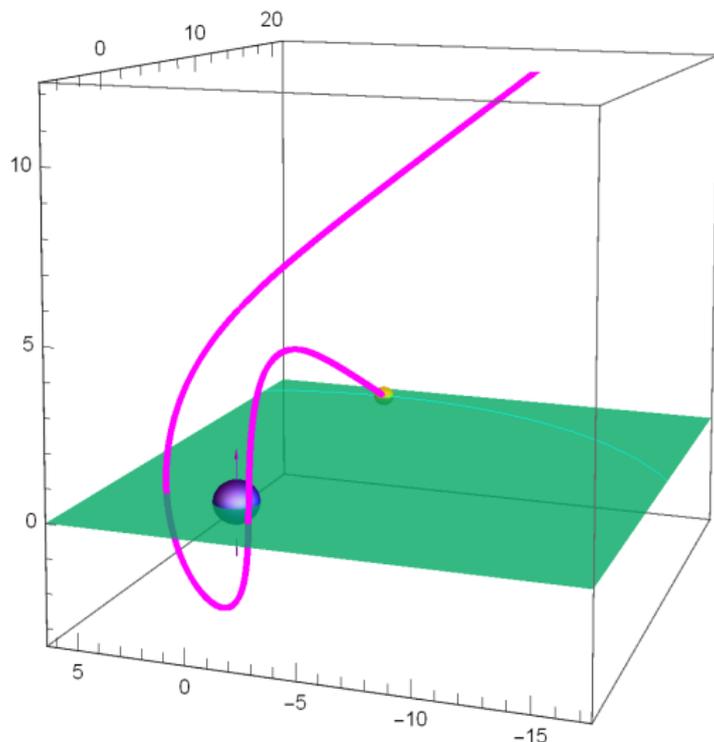
First light echo: 1 intersection of equatorial plane



3D photon trajectory

Second light echo: 2 intersections of equatorial plane

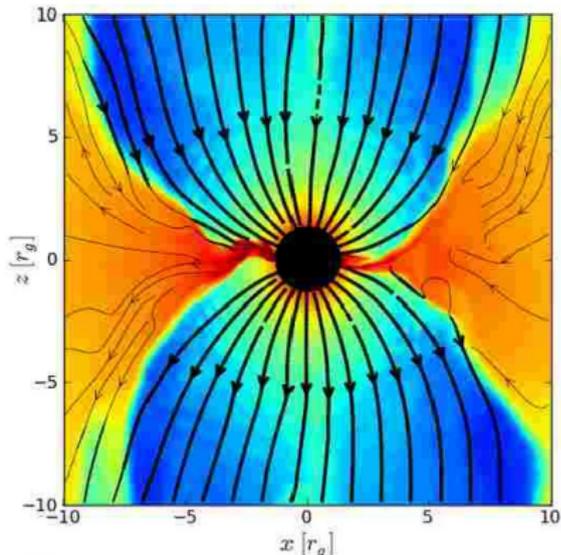
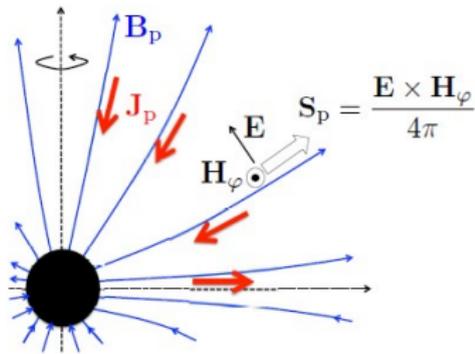
$$\lambda = -1.78, \quad q = 5.2, \quad r_h = 1, \quad r_s = 20, \quad r_{\min} = 3.11$$



Astrophysical Case 2: GRMHD accretion simulation!!!

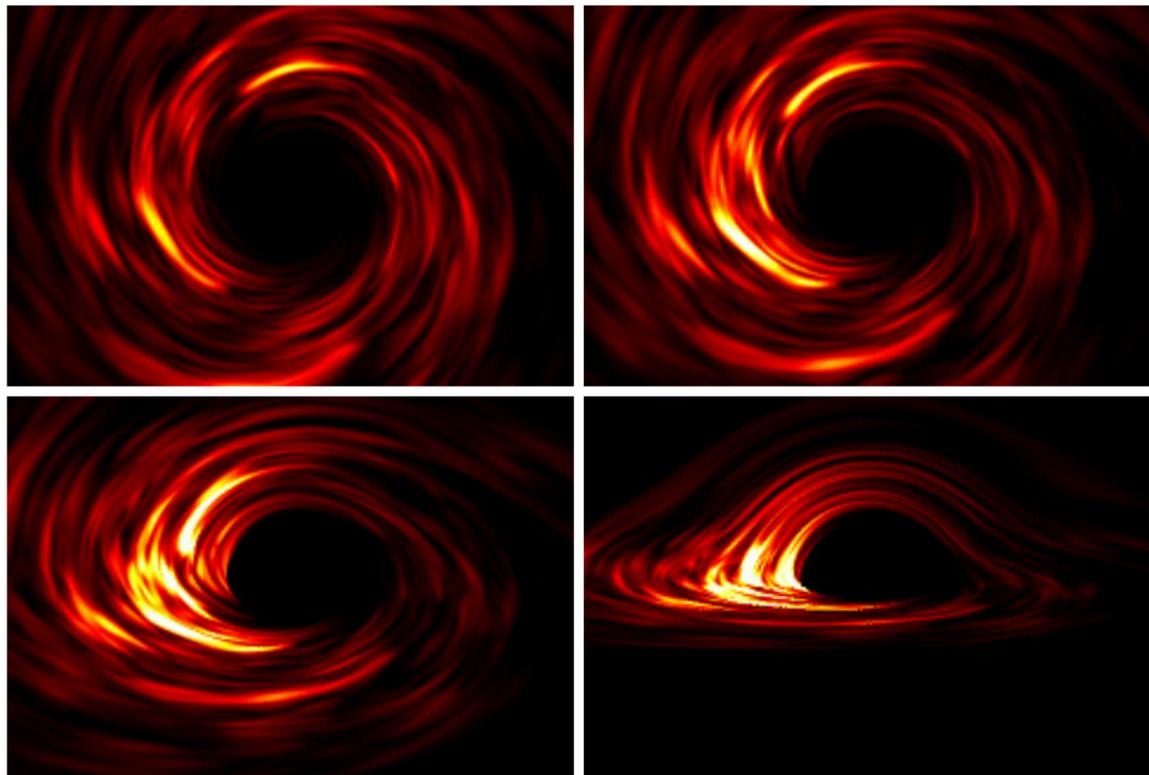
Radiation from both the **outside** and **inside** photon spheres r_{ph}

The Blandford-Znajek process (**quite different from the α -disk!**) is a suitable model for the General Relativistic Magnetohydrodynamics (GRMHD) accretion onto black holes, in which the inflowing plasma is strongly heated even in the vicinity of the event horizon by the radial electric current



Astrophysical Case 2: GRMHD accretion simulation

Radiation from both the **outside** and **inside** photon spheres r_{ph}

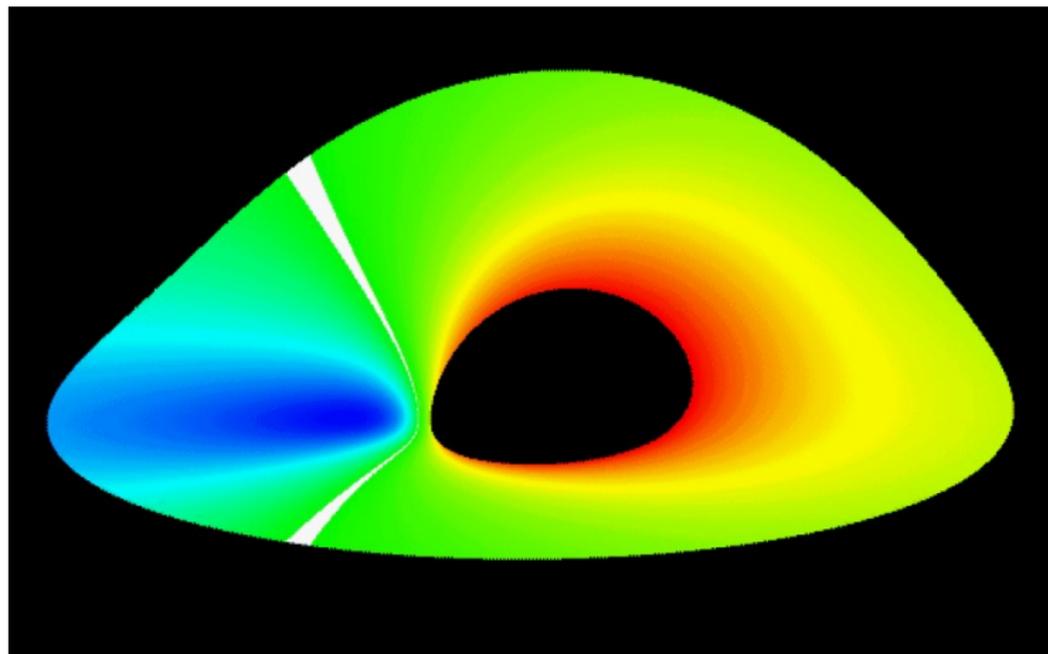


Fe K α line at 6.4 keV

Armitage & Reynolds 2003



Astrophysical Case 2: Line emission from accretion disk
Radiation from both the **outside** and **inside** photon spheres r_{ph}



B.C.Bromley, K.Chen, W.A.Miller ApJ 475 57 (1997)

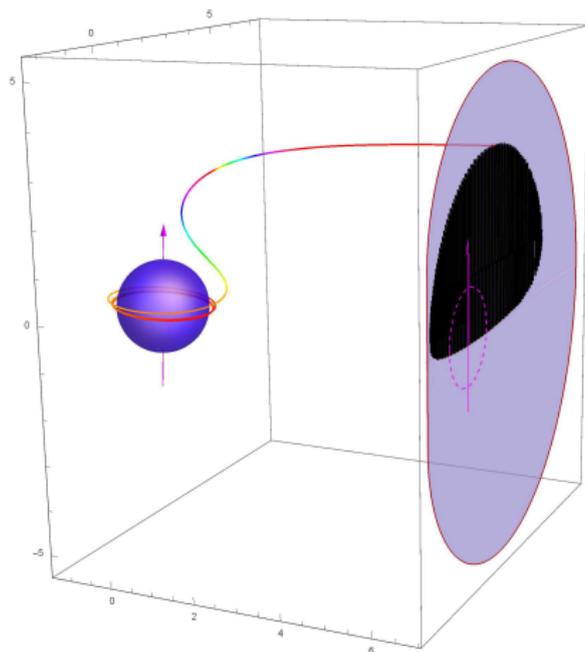
Astrophysical Case 2: Outgoing photon from $r = 1.01r_h$

Radiation **inside** the photon spheres r_{ph}

SgrA*, $a = 1$, $\theta_0 = 82.2^\circ$ — position angle of observer

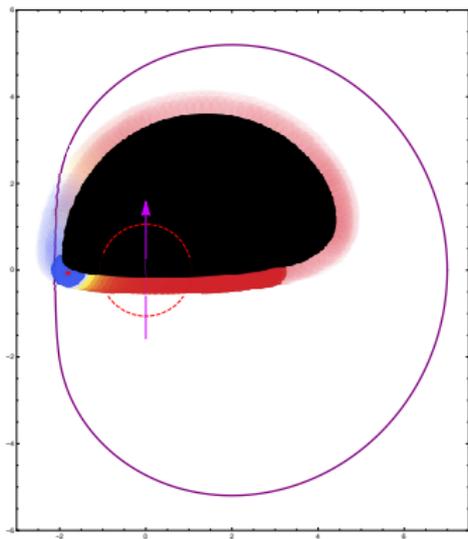
Black hole shadow (black region) is recovered by emission of the nonstationary inner part of accretion disk adjoining the event horizon.

Photon trajectory with impact parameters $\lambda = -1.493$ and $q = 3.629$:



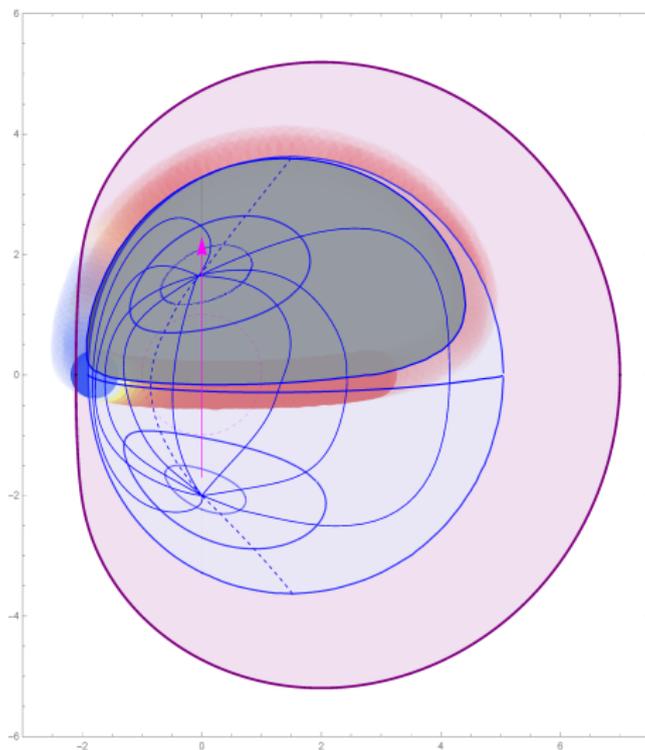
Close to the edge of black hole SgrA^* , $a = 0.998$:

Highly red-shifted radiation of the innermost part of accretion disk in the vicinity of event horizon



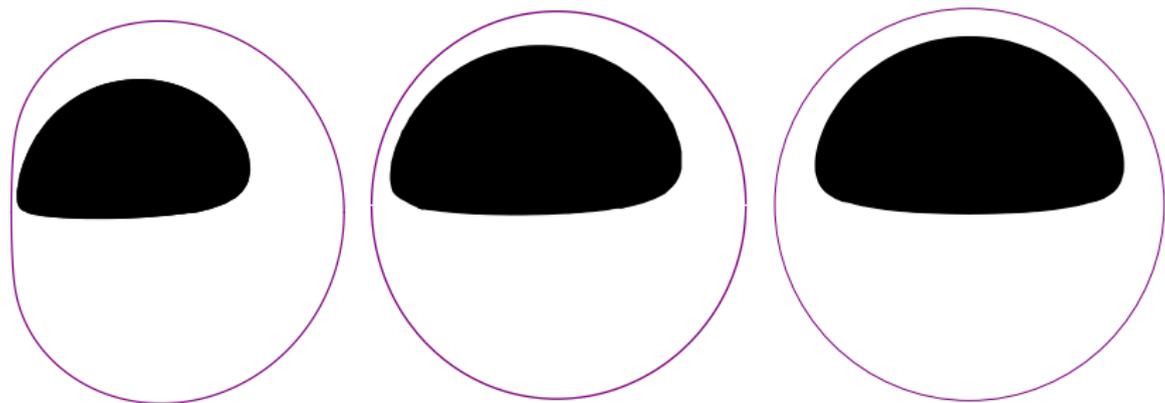
- Red star ★ — brightest point in the thin accretion disk
- Purple curve — boundary of the classical black hole shadow
- Dark region — viewed lensed image of the northern hemisphere of the event horizon globe (boundary of this dark region is the equator at the event horizon globe)

Shadow (silhouette) of the whole horizon globe
Black hole may be viewed from all sides at once!

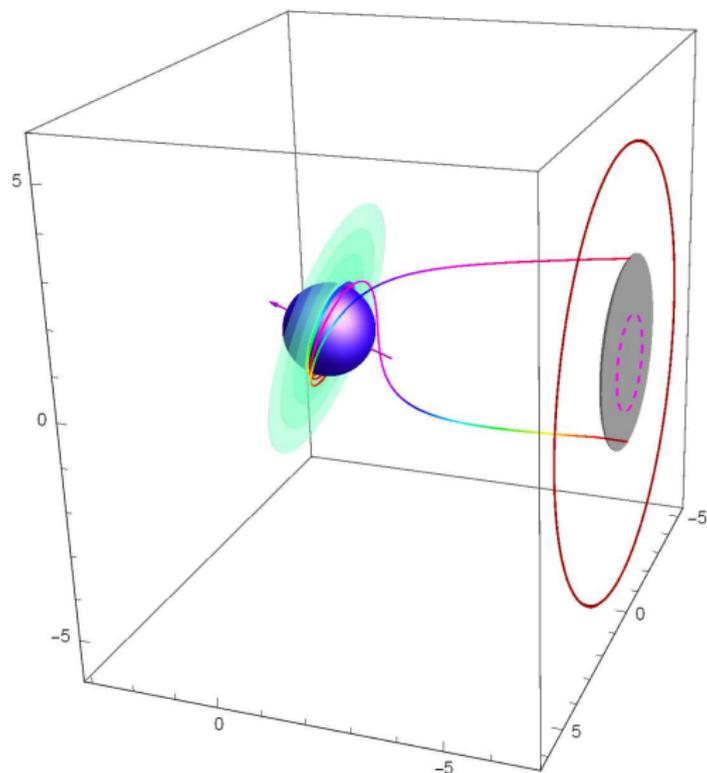


SgrA*, $\theta_0 = 82.2^\circ$: Shadows (silhouettes) of the northern hemisphere of event horizon (black region) projected inside an outline of the black hole shadow (purple closed curves)

Black holes (from left to right) with spin $a = 0.9982, 0.65$ and 0



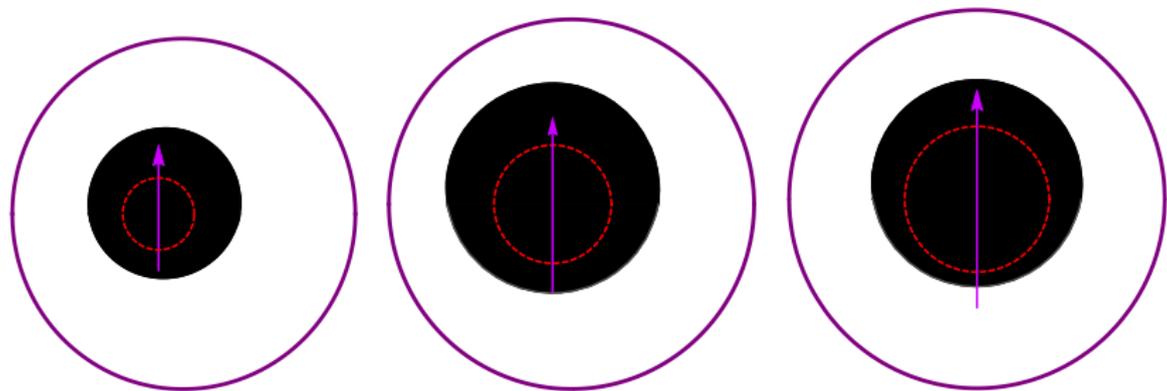
M87*, $\theta_0 = 17^\circ$: Silhouettes of the southern hemisphere of event horizon (gray region) projected inside the black hole shadow (purple closed curves)



Green oval — thin accretion disk around M87*

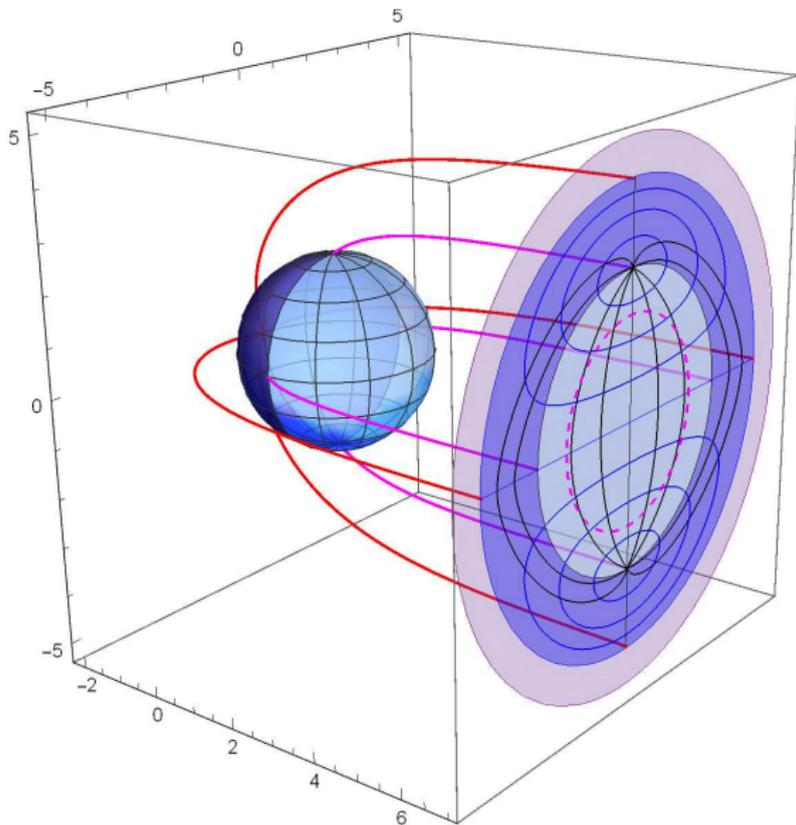
M87*, $\theta_0 = 17^\circ$: Silhouettes of the southern hemisphere of event horizon (black region) projected inside a outline of the black hole shadow (purple closed curves)

Black holes (from left to right) with spin $a = 0.9982, 0.65$ and 0

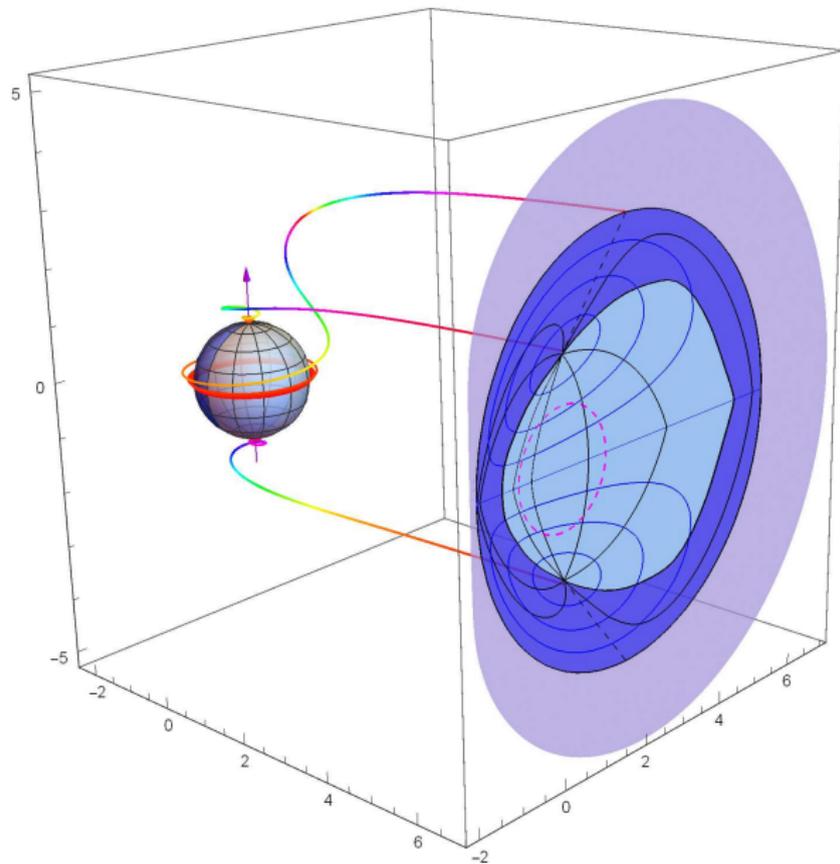


SgrA*, $a = 0$:

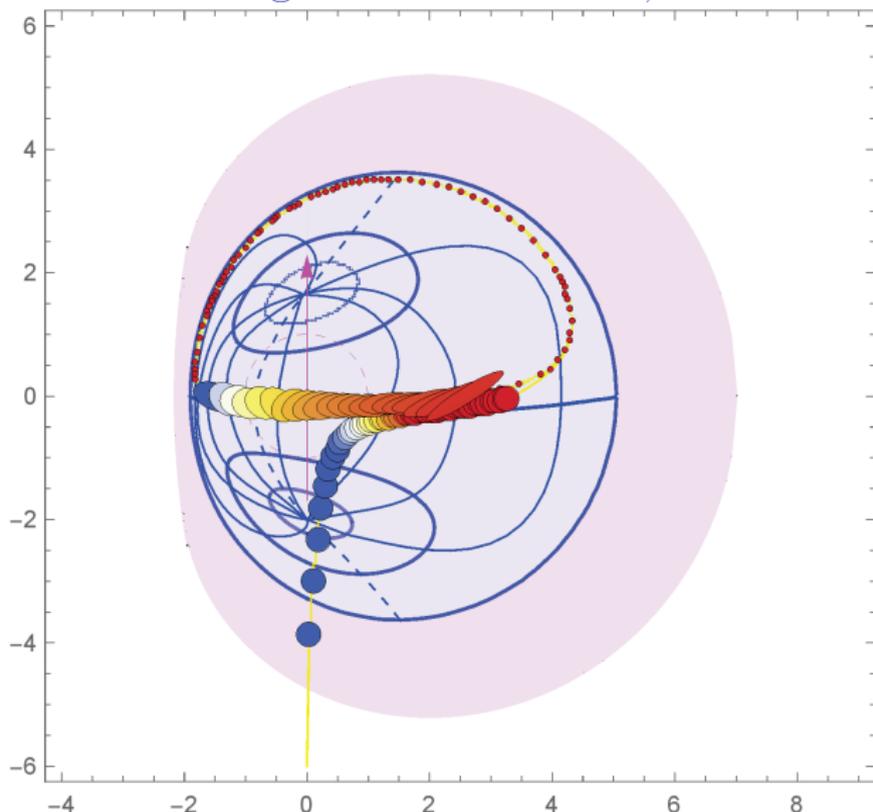
Silhouette of the event horizon globe (dark and light blue regions) projected inside the classical black hole shadow (purple disk)



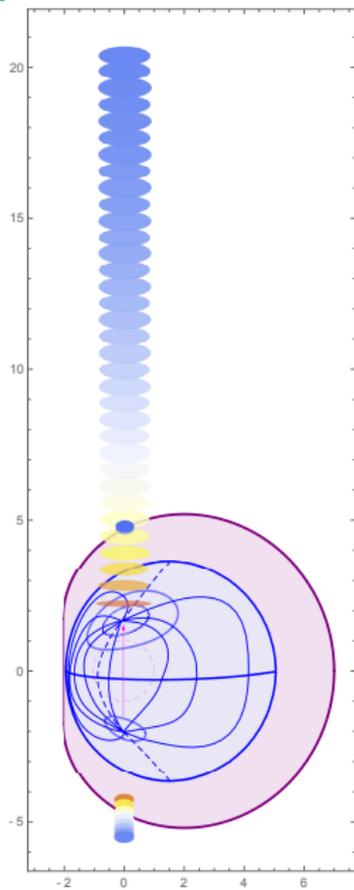
SgrA*, $a = 0.9982$: outgoing photons from $r = 1.01r_h$



SgrA*, $a = 0.9982$: gravitational lensing of the compact emitting source falling into black hole, $\theta_0 = 82.2^\circ$

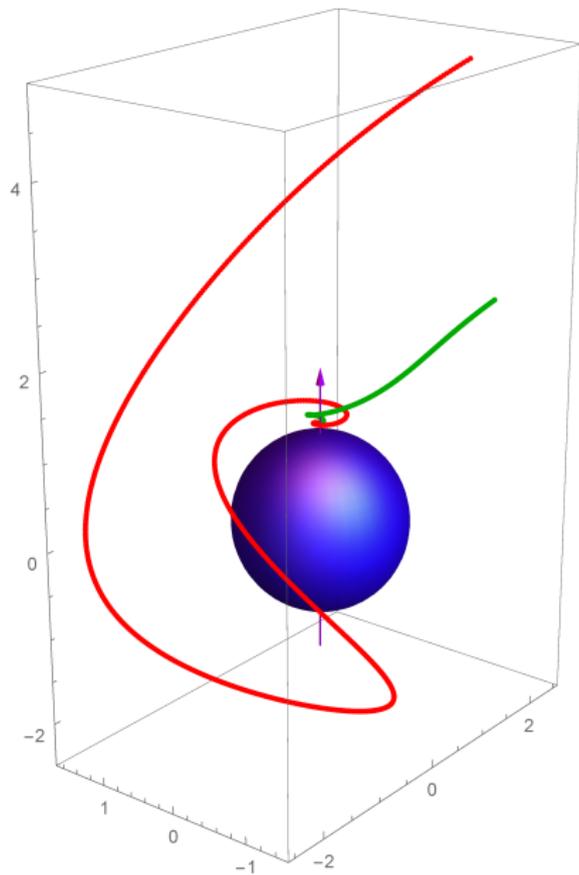


Moving hot spot in the jet from black hole SgrA*: Direct image and 1-st light echoes in discrete time intervals

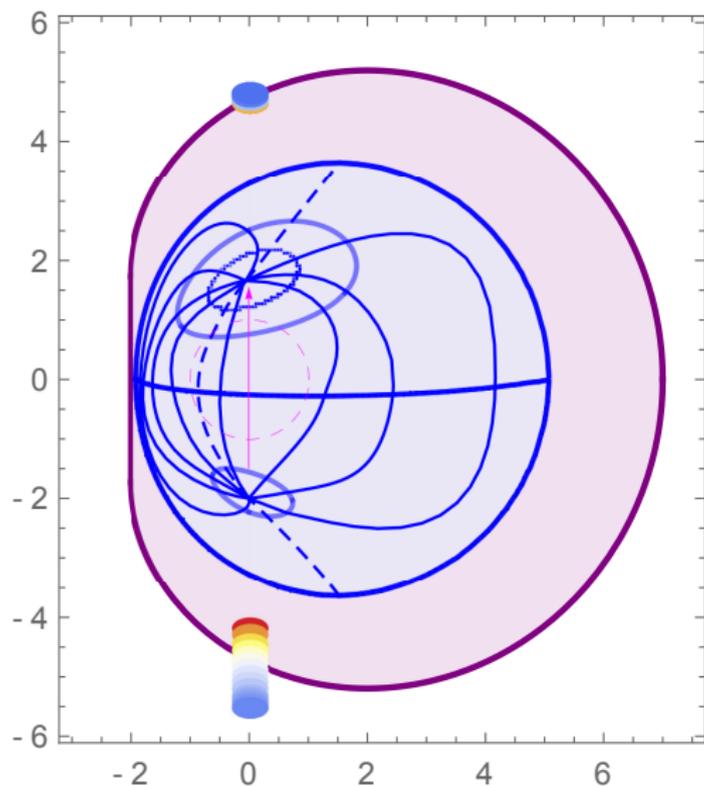


3D photons trajectories, starting at $r = 1.1r_h$

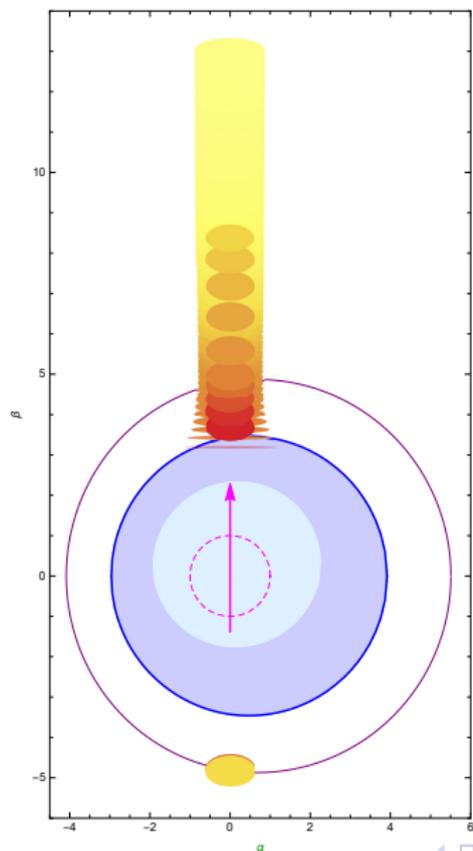
Prime image and second light echo: 2 intersections of equatorial plane



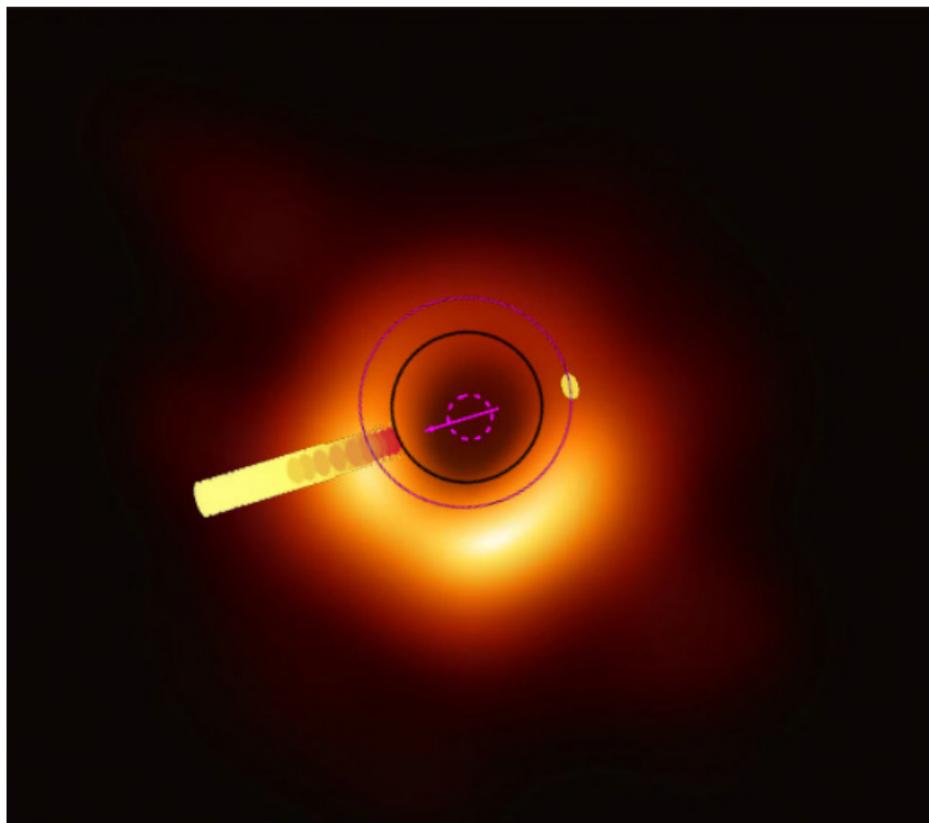
Moving hot spot in the jet from black hole SgrA*:
1-st light echoes near the outlet of the black hole shadow (closed purple curve)



Moving hot spot in the jet from black hole M87*:
1-st light echoes near the outlet of the black hole shadow (closed purple curve)



Direct image and 1-st light echoes of the moving hot spot in the jet from the supermassive black hole M87* in discrete time intervals



Conclusion: black holes are at last discovered !!!



Is it possible to explain this image without black hole?

VD, Nazarova, Smirnov: arXiv:1903.09594 and arXiv:1906.07171

Conclusion and Discussion

(1) **Luminous stationary background behind the black hole:**

The dark classical black hole shadow is viewed, which is a capture photon cross-section in the black hole gravitational field.

(2) **Luminous accretion inflow near the black hole event horizon:**

The dark event horizon shadow is viewed, which is a lensed image of the event horizon globe.

★ The dark silhouette of the event horizon shadow (rather than the classical black hole shadow) is viewed at the image of supermassive black hole M87* obtained by the EHT.

★ The unique information for the verification of gravity theories in the strong field limit will be provided by the Millimetron Space Observatory.

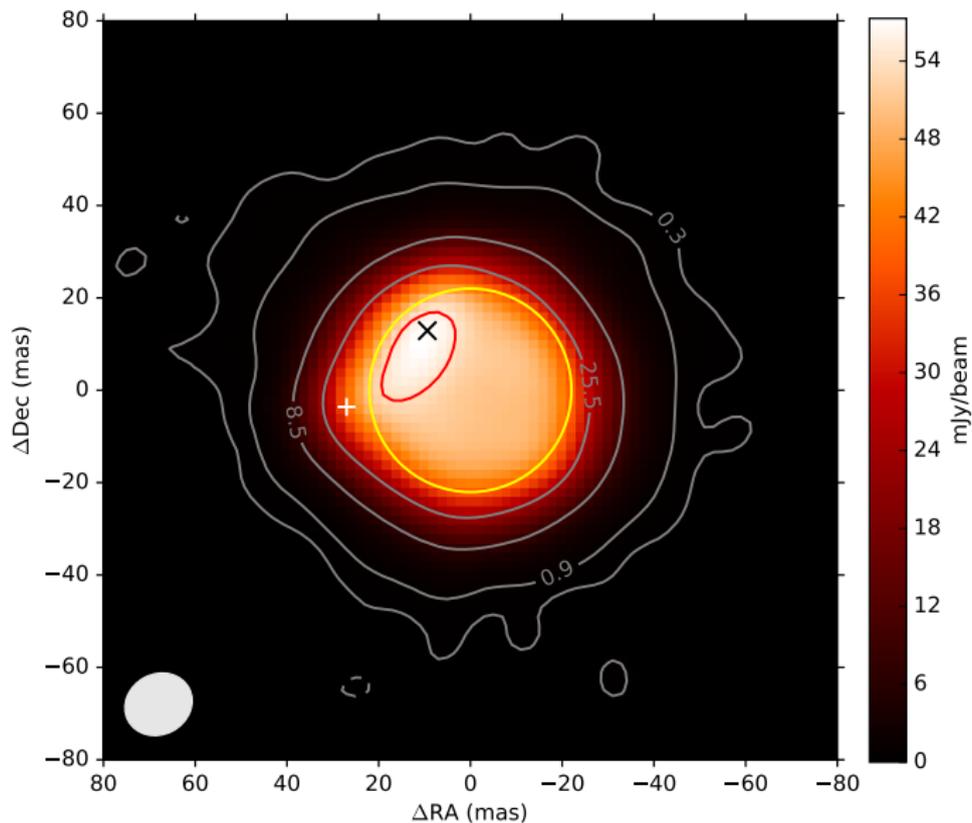
For the details of black hole images see:

1. VD, N.O.Nazarova [Silhouettes of invisible black holes](#) УФН 190 627 (2020); Physics-Uspekhi 63 (6) 583 (2020)
2. VD [To see invisible: image of the event horizon within the black hole shadow](#) Int. J. Mod. Phys. D 28, 1941005 (2019)
3. VD, N.O.Nazarova [Visible shapes of black holes M87* and SgrA*](#) Universe 2020, 6(9), 154
4. VD, N.O.Nazarova, V.P.Smirnov [Event horizon silhouette: implications to supermassive black holes M87* and SgrA*](#) Gen. Relativ. Gravit. (2019) 51: 81
5. VD, N.O.Nazarova [The brightest point in accretion disk and black hole spin: implication to the image of black hole M87*](#) Universe 2019, 5(8), 183
6. VD, N.O.Nazarova [Gravitational lensing of a star by rotating black hole](#) Письма в ЖЭТФ, 106, 609 (2017); JETP Letters, 106, 637 (2017)
7. VD, N.O.Nazarova [Modeling the motion of a bright spot in jets from black holes M87* and SgrA*](#) arXiv:2010.01885

Thanks to all

Image of the object which is not a black hole

ALMA: Star Betelgeuse arXiv:1706.06021

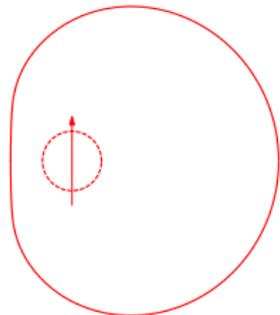


Astrophysical Case 1: Classical black hole shadow

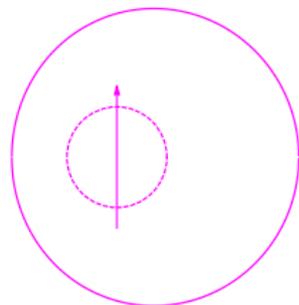
Contour (boundary) of the black hole shadow on the bright background

Radiation outside the photon spheres r_{ph}

Distant observer is in the black hole equatorial plane



$a = 1$



$a = 0.65$

Parametric equation for black hole shadow: $(\lambda, Q) = (\lambda(r), Q(r))$:

$$\lambda = \frac{(3-r)r^2 - a^2(r+1)}{a(r-1)}, \quad q^2 = \frac{r^3[4a^2 - r(r-3)]^2}{a^2(r-1)^2}$$

Bardeen 1973, Chandrasekhar 1983

λ — horizontal and $q = \sqrt{Q}$ — vertical photon impact parameters,

Q — Carter constant, **arrow** — black hole rotation axis

dashed circle — black hole event horizon $r_h = (1 + \sqrt{1 - a^2})$

Equations of motion of a test particle

B. Carter 1968

The Hamilton-Jacobi equation for the Jacobi action S

$$\frac{\partial S}{\partial \lambda} = \frac{1}{2} g^{ij} \left[\frac{\partial S}{\partial x^i} - \epsilon A_i \right] \left[\frac{\partial S}{\partial x^j} - \epsilon A_j \right]$$

If there is a separable solution:

$$S = -\frac{1}{2} \mu^2 \lambda - E u + \Phi \varphi + S_\theta + S_r$$

$$p_\theta = \frac{\partial S}{\partial \theta}, \quad p_r = \frac{\partial S}{\partial r}$$

$$p_\theta^2 + \left(a E \sin \theta + \frac{\Phi}{\sin \theta} \right)^2 + a^2 \mu^2 \cos^2 \theta =$$

$$= \Delta p_r^2 - 2[(r^2 + a^2)E - a\Phi + \epsilon r] p_r + \mu^2 r^2 \quad \Rightarrow \quad = \mathcal{K} = \text{const}$$

$$p_\theta = \frac{dS}{d\theta} = \sqrt{V_\theta}, \quad p_r = \frac{dS}{dr} = \frac{1}{\Delta} \sqrt{V_r}, \quad \Delta = r^2 - 2r + a^2 + e^2$$

Path integral equations of motion

C. T. Cunningham, J. M. Bardeen 1973

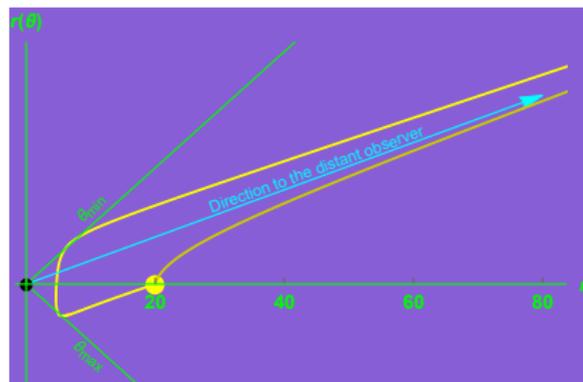
$$\int^{\theta} \frac{d\theta}{\sqrt{V_{\theta}}} = \int^r \frac{dr}{\sqrt{V_r}}, \quad V_{\theta}(\theta_{\min}) = 0, \quad V_r(r_{\min}) = 0$$

The integrals are understood to be path integrals along the trajectory

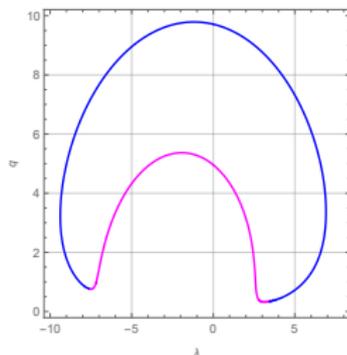
Integral equation with respect to $\lambda = \Phi/E$ and $q = Q^{1/2}/E$

for the trajectories of the first light echo:

$$\int_{\theta_s}^{\theta_{\max}} \frac{d\theta}{\sqrt{V_{\theta}}} + \int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\sqrt{V_{\theta}}} + \int_{\theta_0}^{\theta_{\min}} \frac{d\theta}{\sqrt{V_{\theta}}} = \int_{r_s}^{r_{\min}} \frac{dr}{\sqrt{V_r}} + \int_{r_{\min}}^{r_0} \frac{dr}{\sqrt{V_r}}$$



2D photon trajectory $r(\theta)$



Solutions (λ, q)

Заключение: чёрные дыры наконец-то обнаружены !!!



Темный силуэт — линзированное изображение южной
полусферы горизонта событий

Контур темного силуэта — линзированное изображение
экватора горизонта событий

Утверждения и Выводы:

★ Классическая тень черной дыры наблюдается в случае удаленного от черной дыры фона (расположенного вне световых сфер)

★ Тень горизонта событий видна в случае яркого фона вблизи горизонта событий черной дыры (расположенного внутри световых сфер)

★ На первом опубликованном изображении черной дыры в M87 наблюдается темная тень горизонта событий, но не классическая тень черной дыры

★ Аккреционная яркость черных дыр может значительно превышать яркость фона в виде звезд и облаков газа. По этой причине возможность наблюдения классической тени черной дыры с мощной аккреционной светимостью представляется маловероятной.